

Consider a complete set of all such pairwise binary preferences between alternatives in  $X$ . The following two properties might be reasonably assumed about “rational” preferences:

- **Asymmetry** If  $x$  is strictly preferred to  $y$ , then  $y$  is not strictly preferred to  $x$ .
- **Negative transitivity** If  $x$  is not strictly preferred to  $y$  and  $y$  is not strictly preferred to  $z$ , then  $x$  is not strictly preferred to  $z$ .

Asymmetry and negative transitivity can be considered as “minimal consistency properties” for an expression of preference among a set of alternatives. A binary relation  $\succ$  on a set  $X$  is called a *preference relation*, if it is asymmetric and negatively transitive. While asymmetry is quite plausible, negative transitivity is not a completely innocuous assumption, as illustrated by the following example:

**Example E.1** Suppose you are choosing among jobs in three different cities. Suppose the two factors that matter most to you are income and the climate. The job in city  $x$  has a high salary of \$100,000, and the climate is average. The job in city  $y$  offers a salary of only \$50,000, but the climate is terrific. The job in city  $z$  offers a moderate salary of \$70,000 and the climate is poor. You might not strictly prefer  $x$  to  $y$  because although  $x$  offers a great salary,  $y$  offers a great climate. Likewise, you might not strictly prefer  $y$  to  $z$  because again, while  $y$  offers a great climate,  $z$  offers a higher salary. However, you may very well prefer  $x$  to  $z$ , since  $x$  has both a higher salary and a better climate than does  $z$ . These preferences would violate negative transitivity.

Despite such shortcomings, the properties of asymmetry and negative transitivity form the classical basis for modeling customer preferences. The following are some examples of preference relations:

**Example E.2** (LEXICOGRAPHIC MODEL) This model of preferences, due to Tversky [521], assumes customers rank order various attributes of a product and then evaluate them using a lexicographic rule. For example, a tennis racquet comes in three models  $A$ ,  $B$ , and  $C$  with the following features:

<i>Product</i>	<i>Wide Body?</i>	<i>Graphite?</i>	<i>Black?</i>
<i>A</i>	Yes	No	Yes
<i>B</i>	Yes	Yes	No
<i>C</i>	No	Yes	Yes

The customer’s decision rule is to rank all attributes from most important to least important and then eliminate alternatives which do not possess the most important attributes. If more than one alternative remains, the next most important attribute is chosen as a criterion for elimination of alternatives, and so on.

For example, a customer may care most about whether a racquet has a wide body, then whether it is graphite, and lastly whether it is black. He would then prefer racquets with a wide body to all others without a wide body (regardless of the other attributes). Among all those with wide bodies, he would then select those that have graphite construction; among the remaining, he may select only the ones that are black, and so on. So for our three products above, this customer would prefer product in the order  $B$ ,  $A$ ,  $C$ . One can verify that the lexicographic model generates a preference relation among the alternatives.